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DTU

Application of asymptotic speed deficit concept to existing engineering wake model

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WAKE DECAY FOR THE INFINITE WIND TURBINE ARRAY

Application of asymptotic speed deficit concept to existing engineering wake model

Outline

- Background
- Asymptotic speed deficit from boundary layer considerations
- “WAsP Park” model details
- Asymptotic speed deficit of the “WAsP Park” model
- Adjustment of WAsP Park model
- Comparative wind farm predictions
- Conclusions

Background

Very large wind farms:

- Standard wake models seems to underpredict wake effects.

Recent investigations by Sten Frandsen [1, 2]:

- The reason is the lack of accounting for the effect a large wind farm may have on the atmospheric boundary layer, e.g. by modifying the vertical wind profile.

- In some way the effect of an extended wind farm resembles that of a change in surface roughness: increased equivalent roughness length.

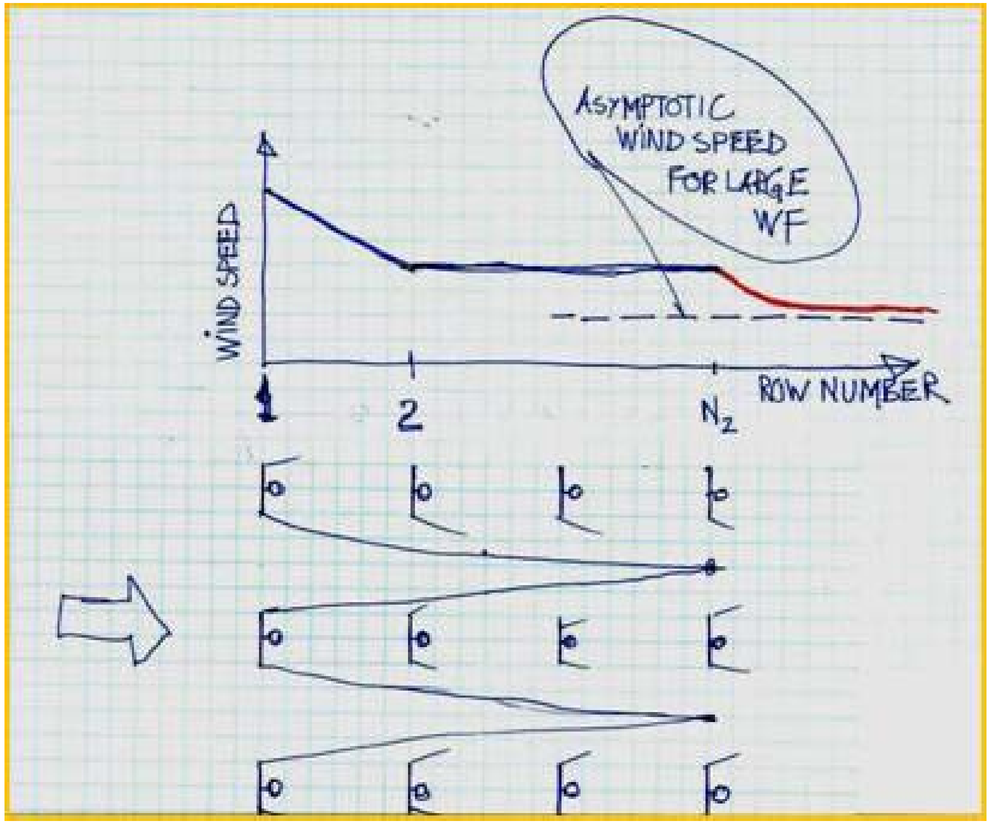
Idea:

- While more detailed models are underway [3],
  modify the existing WAsP Park engineering wind farm wake model to take this boundary-layer effect into account.

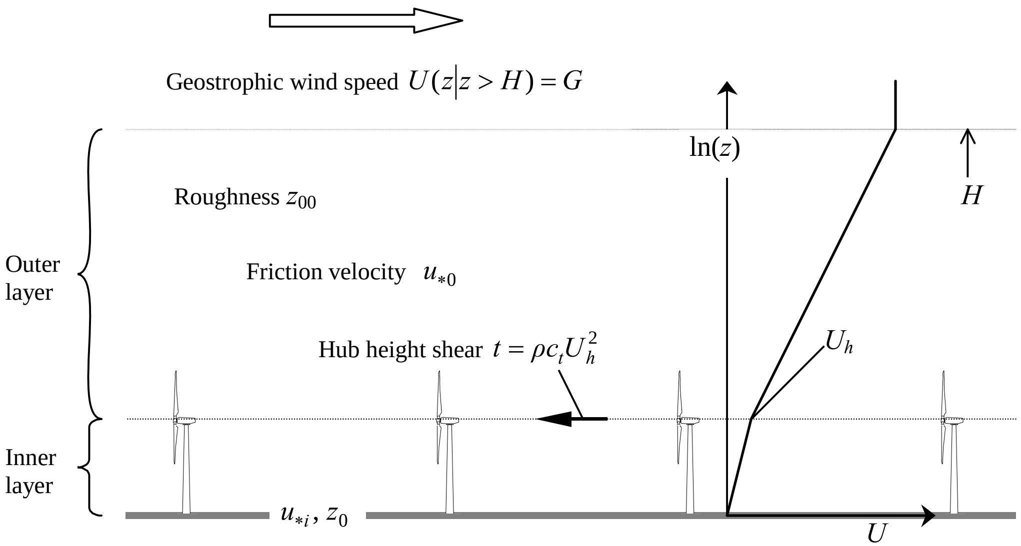
[1] Frandsen, S.T., Barthelmie, R.J., Pryor, S.C., Rathmann, O., Larsen, S., Højstrup, J. and M. Thøgersen , Analytical modelling of wind speed deficit in large offshore wind farms. Wind Energ. 2006, 9: 39-54.
[2] Frandsen, S: The wake-decay constant for the infinite row of wind turbine rotors. Draft paper (2009).
[3] Rathmann O., Frandsen S, Barthelmie R., Wake Modelling for intermediate and large wind farms, Paper BL3.199, EWEC 2007

When should a wind farm be considered as large/infinite?

(Hand drawing illustrating the initial idea)



BL-Limited infinite wind farm



Jump in friction velocity at hub-height due to rotor thrust: $\rho(u_*^{eff})^2 = \rho(u_{*i})^2 + t$

Approximation: homogeneously distributed thrust c_t

$c_t = \pi/8\ C_t/(s_r\ s_f)$, $t = \rho C_t U_h^2$

s_r and s_f: dimensionless* WTG-distances (along- and across-wind) *by D_{rotor}

Z<h: profile according to ground surface friction velocity u_{*i} / roughness z_0.

Z>h: profile according to increased friction velocity u_*^{eff}(= u_{*0}) / roughness z_0^{eff} (=z_{00}).

Equivalent, effective surface roughness: $z_0^{eff} = h_H \cdot \exp\left(-\kappa / \sqrt{c_t + \left(\kappa / \ln(h_H / z_0)\right)^2}\right)$

Asymptotic speed deficit from boundary layer considerations (3)

Approximate geostrophic drag-law
$$G \approx \frac{u_*}{\kappa}\left(\ln\left(\frac{G}{fz_0}\right) - A_*\right)$$

General hub-height wind speed:
$$U(h) = \frac{G}{1+\left(\ln\frac{G}{h\cdot f} - A_*\right)i}$$

Free flow: $i_0 = 1/\ln\frac{h}{z_0}$ Flow over wind farm: $i_{Tot} = \sqrt{i_0{}^2 + i_{add}{}^2}, \quad i_{add} = \frac{\sqrt{c_t}}{\kappa}$

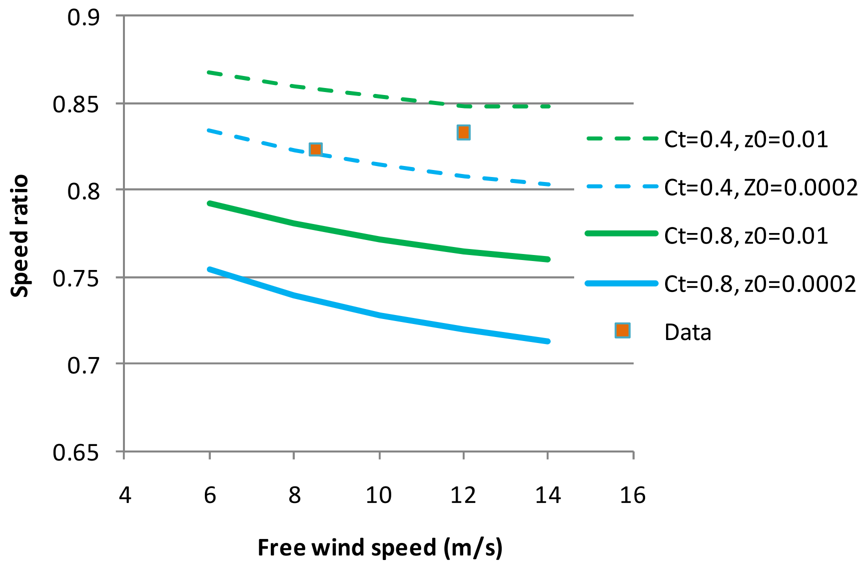
Relative speed deficit ε:
$$1-\varepsilon = \frac{1+\ln\left(\frac{G}{h\cdot f'}\right)i_0}{1+\ln\left(\frac{G}{h\cdot f'}\right)i_{Tot}}$$

Asymptotic speed deficit from boundary layer considerations (3)

Comparison with wind farm (Horns Rev):

$s_r \approx s_f \approx 7$, h=80m, D_R = 60 m

Wake deficit about 50% of the BL-limiting value.
Horns Rev wind farm NOT "infinite".

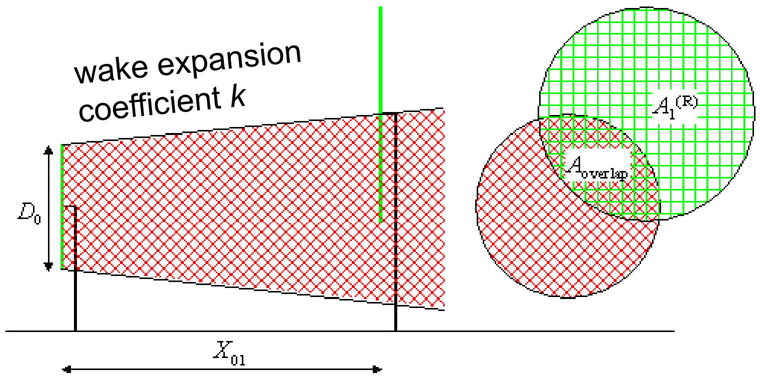


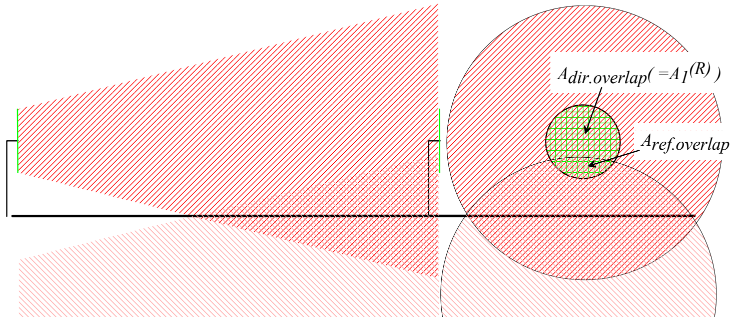
Horns Rev	
Distance for severe wake interference (k_{wake}=0.075)	Actual extension
7.5 km	5 km

Power density (W/m²) [4]	
Horns Rev 2MW turb's (observed)	Entire North Sea 5 MW turb's (Frandsen – BL-limited)
2.9	1.8

[4] Barthelmie,R.J., Frandsen,S.T., ., Pryor, S.C., Energy dynamics of an infinitely large offshore wind farm., Paper 124, European Offshore WInd 2009 Conference and Exhibition, Stockholm, Sweden (Sept. 2009).

Wake Evolution and speed deficit [5,6]





Speed deficit from single wake:

$$\delta V_{01}^{(type)} = U_0\left(1-\sqrt{1-C_t}\right)\left(\frac{D_0}{D_0+2kX_{01}}\right)^2 \frac{A_{(type)overlap}}{A_1^{(R)}}, \quad (type) = "dir.", \quad "ref."$$

Resulting speed deficit at a downwind turbine:

$$\delta V_{turb}^2 = \sum_{i \in upw.turb's} \left((\delta V_{i,turb}^{(dir.)})^2 + (\delta V_{i,turb}^{(ref.)})^2 \right)$$

[5] N.O.Jensen, A Note on Wind Generator Interaction, Risø-M-2411, Risø National Laboratory 1983.
[6] I.Katic, J.Højstrup, N.O.Jensen, A Simple Model for Cluster Effeciency, Paper C6, EWEC 2006, Rome, Italy, 1986.

Speed deficit for a turbine in an infinite wind farm

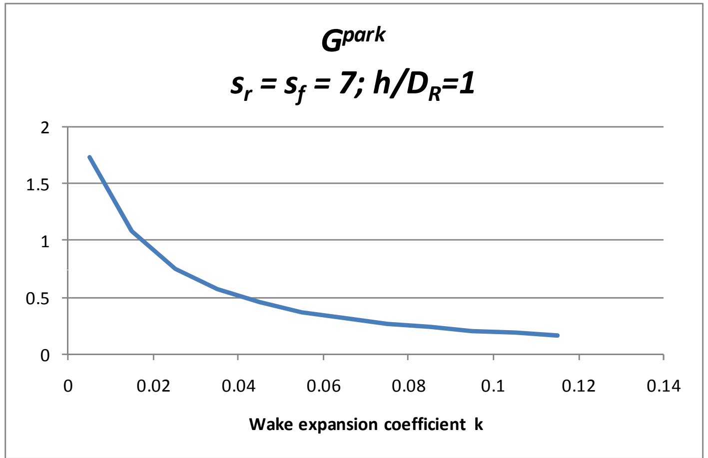
Speed deficit the same for all turbines, thus also the turbine thrusts.

Infinite (convergent!!) sum:

$$(\delta V)^2 = (U_{upwind}\varepsilon_0)^2 \sum_{j=1}^{\infty} N(s_j)\varepsilon_w(x_j)^2; \quad \varepsilon_w(x) = \left(\frac{D_R}{D_R + 2kx}\right)^2; \quad \varepsilon_0 = \left(1 - \sqrt{1 - C_t}\right)$$

x_j: Distance to upwind turbine row j. $N(x_j)$: number of turbines row ***j*** throwing wake on the rotor in focus.

U_{upwind}: Wind speed immediately upwind of a turbine



The infinite sum may be approximated by an infinite integral - a simple function *G*:

$$\frac{\delta V}{U_{upwind}\varepsilon_0} = G^{Park}\left(k; s_r, s_f, h/D_R, C_t\right)$$

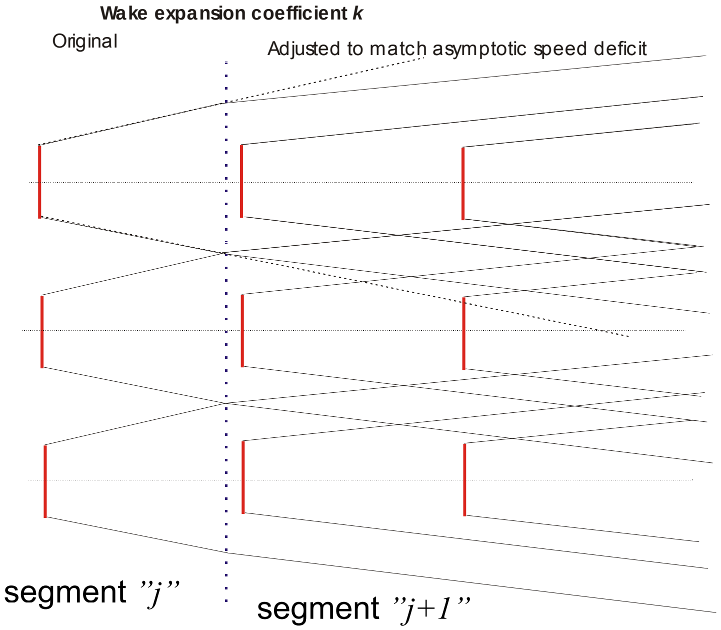
Since $U_{upwind} = U_w = U_{free} - \delta V$:

$$\boxed{\frac{\delta V}{U_{Free}} = \varepsilon_w = \frac{\varepsilon_w^{app}}{1 + \varepsilon_w^{app}}; \quad \varepsilon_w^{app} = \varepsilon_0 G^{park}(layout; k)}$$

Adjustment to match the BL-based asymptotic speed deficit

For “deep” positions the wake expansion coefficient *k* of the Park Model is modified to approach the BL-based asymptotic speed deficit value k_{inf}:

$$\delta V_{infin.park}(k_{inf};[s_r, s_f, h, C_t]) = \delta V_{BL-based}(s_r, s_f, h, C_t)$$



The *k*-change applies when a wake overlaps with a downwind rotor (to both wakes involved), using a relaxation factor F_{relax}:

$$k_{j+1}^{adj} = k_j^{adj} + (k_{inf} - k_j^{adj}) \frac{A_{overlap}}{A_w} F_{relax}$$

The change of the wake expansion cofficient is indicated.

Model-paramters used in the following (based on Horns Rev data):

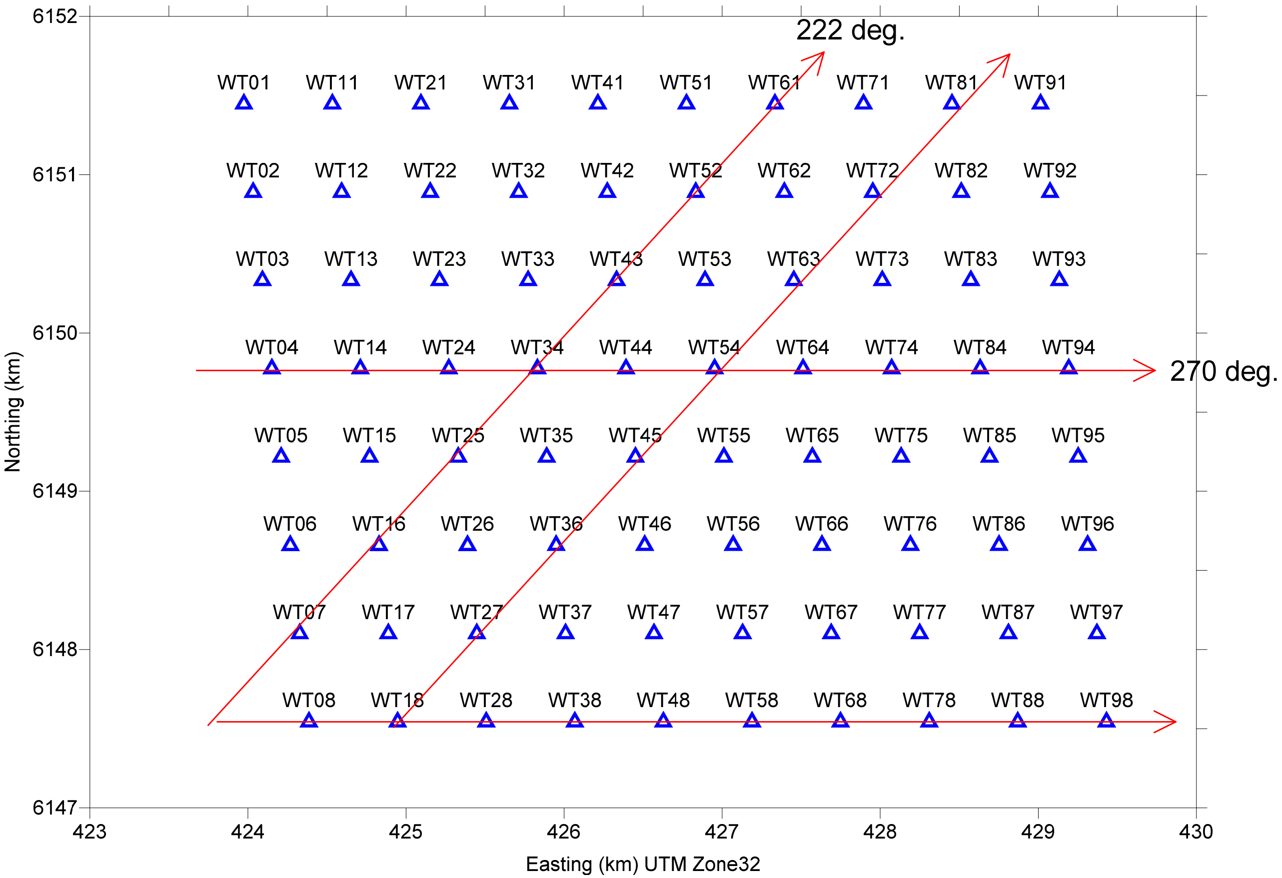
$k_{initial}$:= 0.075 (recommended value for onshore!)

$F_{relax} = 0.2$

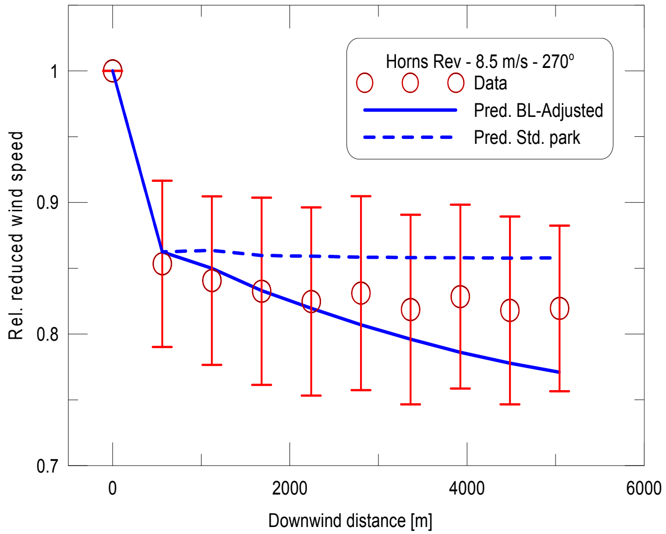
Comparative wind farm predictions: Horns Rev (1)

Turbines: 2MW, D_R = 80m, H_{hub} = 60m

Layout: $s_r = s_f = 7$

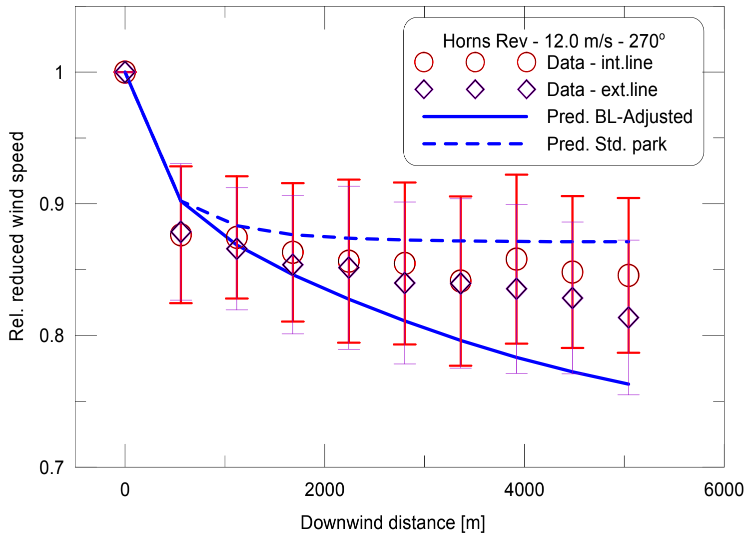


Comparative wind farm predictions: Horns Rev (2)



Wind direction: 270° +/- 3°

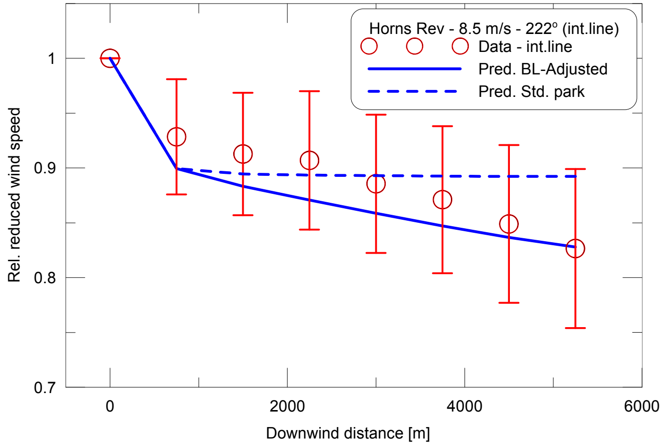
Wind speed: 8.5 m/s +/- 0.5 m/s

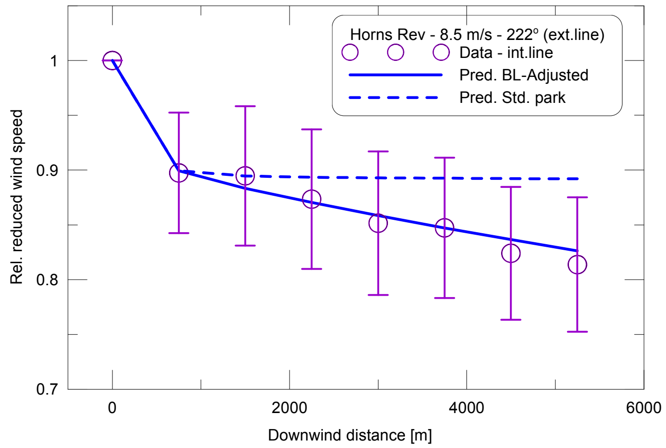


Wind direction: 270° +/- 3°

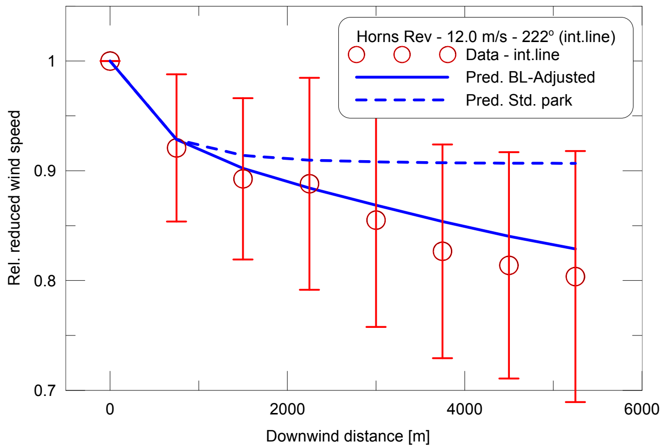
Wind speed: 12.0 m/s +/- 0.5 m/s

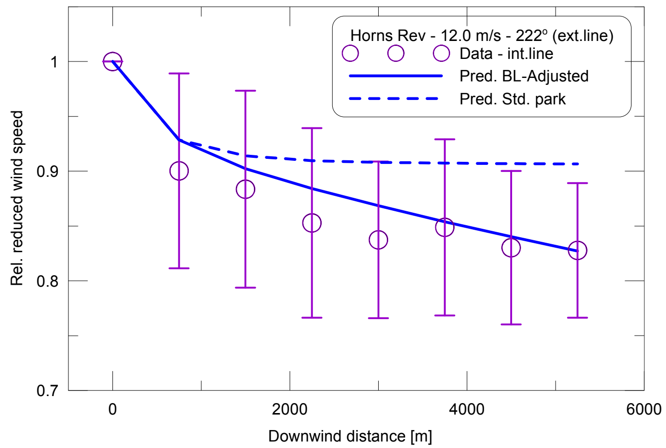
Comparative wind farm predictions: Horns Rev (3)





Wind direction:
222^o +/- 3^o
Wind speed:
8.5 m/s +/- 0.5 m/s



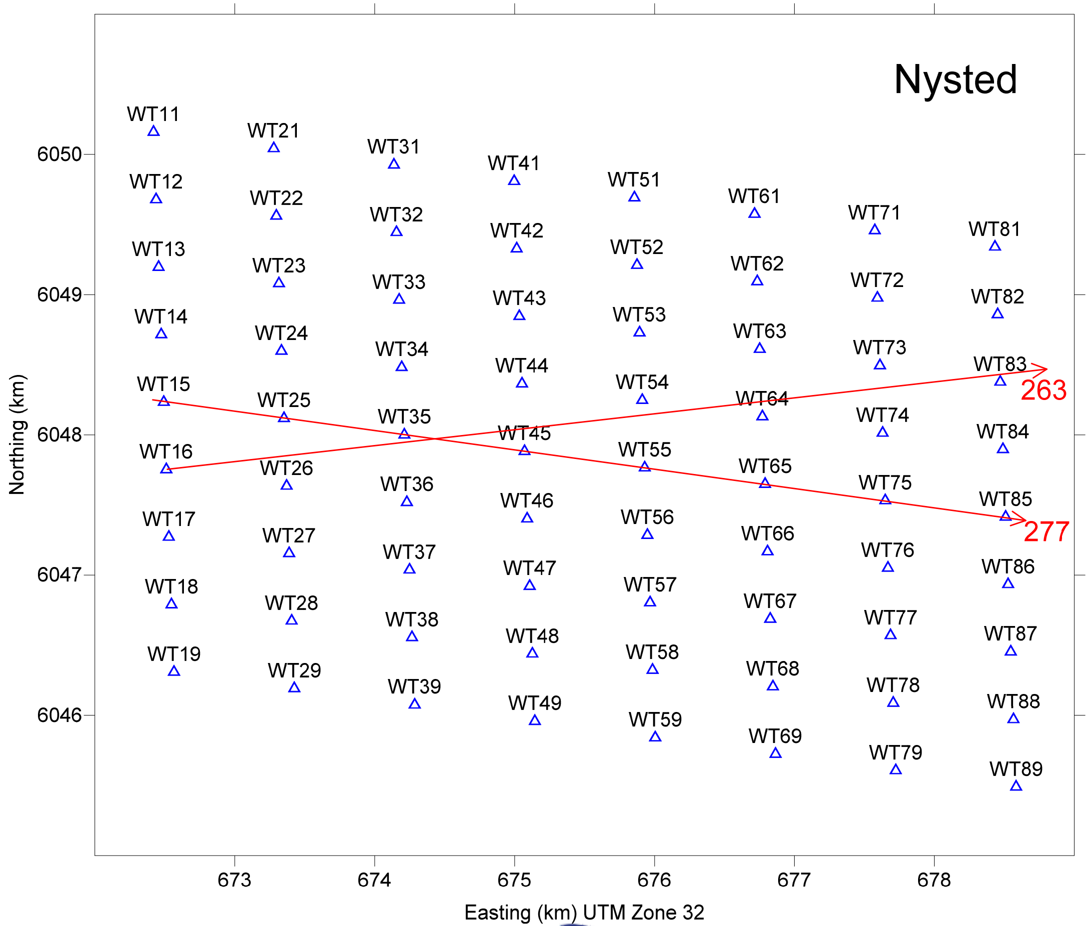


Wind direction:
222^o +/- 3^o
Wind speed:
12.0 m/s +/- 0.5 m/s

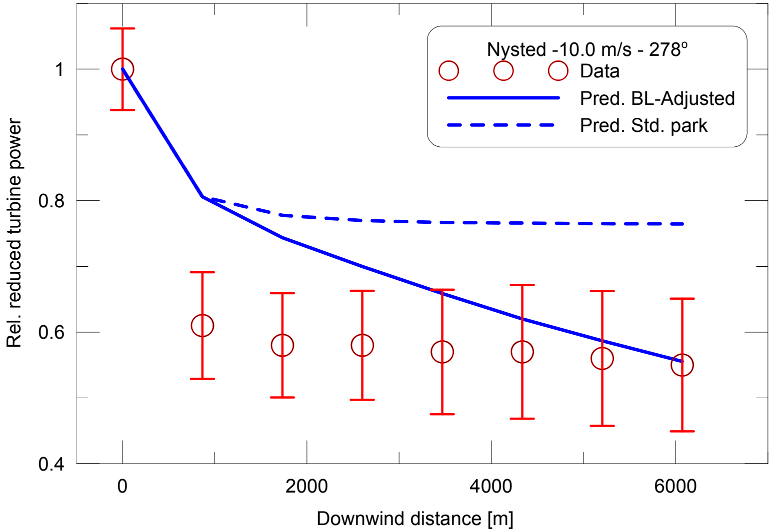
Comparative wind farm predictions: Nysted (1)

Turbines: 2.33 MW, D_R = 82m, H_{hub} = 69m

Layout: s_r = 10.6, s_f = 5.9

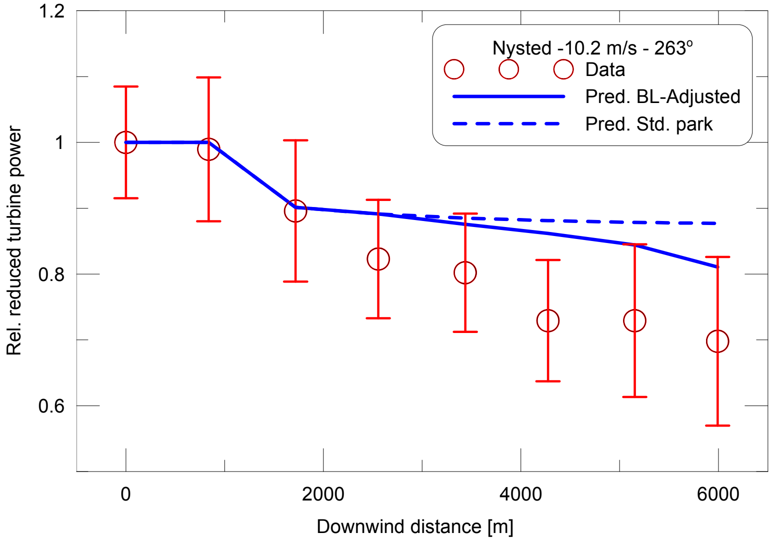


Comparative wind farm predictions: Nysted(2)



Wind direction: 278° +/- 2.5°

Wind speed: 10.0 m/s +/- 0.5 m/s



Wind direction: 263° +/- 2.5°

Wind speed: 10.2 m/s +/- 0.5 m/s

Conclusions

- The adjustment of the wake expansion coefficient towards a value matching the BL-limited asymptotic speed deficit seems a valuable engineering approach
- A value for the wake expansion coefficient close to that normally used for onshore – locations seems reasonable in this approach also for off-shore wind farms
- The model (relaxation factor) needs to be fine-tuned in order not to produce over estimations.
- The model needs to be tested on situations with wake effects between neighboring wind farms.